British Standard

A single copy of this British Standard is licensed to

Ekrem ARSLAN

14 March 2004

This is an uncontrolled copy. Ensure use of the most current version of this document by searching British Standards Online at bsonline.techindex.co.uk

Guide to quality control and performance improvement using qualitative (attribute) data —

Part 3: Technical aspects of attribute charting: special situation handling

ICS 03.120.30

British Standards

Committees responsible for this British Standard

The preparation of this British Standard was entrusted to Technical Committee SS/4, Statistical process control, upon which the following bodies were represented:

Association for Road Traffic Safety and Management (ARTSM)
BAE Systems
British Standards Society (BSS)
Clay Pipe Development Association (CPDA)
Federation of Small Businesses (FSB)
Gauge and Tool Makers Association (GTMA)
General Domestic Appliances Ltd.
Institute of Quality Assurance
National Physical Laboratory
Royal Statistical Society

This British Standard was published under the authority of the Standards Policy and Strategy Committee on 31 October 2003

First published as BS 5701 February 1980

The following BSI references relate to the work on this British Standard:
Committee reference SS/4
Draft for comment 02/400887 DC

ISBN 0 580 42736 6

Amendments issued since publication

Amd. No.	Date	Comments

Contents

Foreword

BS 5701-3:2003 partially supersedes BS 5701:1980 and BS 2564:1955 and all four parts of BS 5701 together supersede BS 5701:1980 and BS 2564:1955, which are withdrawn.

Qualitative data can range from overall business figures such as percentage profit to detailed operational data, such as percentage absenteeism, individual process parameters and product/service features. The data can either be expressed sequentially in yes/no, good/bad, present/absent, success/failure format, or as summary measures (e.g. counts of events and proportions). For measured data control charting refer to BS 5702-1. The focus throughout the BS 5701 family of standards is on the application of attribute control charts to monitoring, control and improvement. The roles of associated, mainly pictorial, diagnostic, presentation and performance improvement tools, such as priority (Pareto) diagrams, cause and effect diagrams and flow charts are also indicated.

This aim of BS 5701 is to be readily comprehensible to the very extensive range of prospective users and so facilitate widespread communication, and understanding, of the method. As such, it focuses to the greatest extent possible on a practical non-statistical treatment of the gathering and charting of qualitative data.

BS 5701-1 demonstrates the business benefits, and the versatility and usefulness of a very simple, yet powerful, pictorial control chart method for monitoring and interpreting qualitative data. In BS 5701-1, the treatment of charting of qualitative data is essentially at appreciation level. However, it is intended to provide adequate information for a gainful first application, by a typical less statistically inclined user, in many everyday situations.

BS 5701-2 continues to focus on the application of standard attribute charts to the monitoring, control and improvement of business processes. This is done at a technical level more suitable for practitioners.

BS 5701-3 concentrates on the statistical basis of, and technical rationale for, attribute control charting. It also gives guidance on dealing with special situations.

BS 5701-4 deals with measuring and improving the quality of decision making in the classification process itself.

A British Standard does not purport to include all the necessary provisions of a contract. Users of British Standards are responsible for their correct application.

Compliance with a British Standard does not of itself confer immunity from legal obligations.

Summary of pages

This document comprises a front cover, an inside front cover, pages i and ii, pages 1 to 31 and a back cover.

The BSI copyright notice displayed in this document indicates when the document was last issued.

1 Scope

BS 5701-3 describes the technical and statistical foundations of attribute control charting. It also gives guidance on dealing with special situations.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

BS EN ISO 9000:2000, Quality management systems — Fundamentals and vocabulary.

BS ISO 3534-1, Statistics — Vocabulary and symbols — Part 1: Probability and general statistical terms.

BS ISO 3534-2, Statistics — Vocabulary and symbols — Part 2: Applied statistics.

3 Terms, definitions and symbols

For the purposes of this part of BS 5701, the terms, definitions and symbols given in BS ISO 3534-1, BS ISO 3534-2 and BS EN ISO 9000:2000, Clause **3** apply with one exception.

The exception is intended for purposes of simplicity. It relates to the symbols for population parameters, sample statistics and their realized values. It is standard statistical practice for population parameters to be symbolized by lower case Greek letters in italics. For example, mean and standard deviation are denoted by μ and σ. A population parameter is a summary measure of the value of some characteristic of a population.

A sample statistic is a summary measure of some observed value of a sample. Sample statistics are symbolized by upper case Latin letters in italics. For example, mean and standard deviation are denoted by $\bar{X}$ and S.

Realized values of sample statistics to be symbolized by lower case Latin letters in italics. For example, mean and deviation are denoted by $\bar{x}$ and s.

Upper case Latin letters in italics are used in Annex A and its applications.

4 Qualitative data fundamentals

4.1 General

Two general classifications of data are quantitative data and qualitative data. This standard focuses on qualitative data. Qualitative data are divided for convenience, into two categories, *classified data* and *count data.*

4.2 Classified data and the binomial distribution

With classified data, each item of data is classified as being one of a number of categories. Frequently the number of categories is two, i.e. a binary situation where, for instance, results are usually expressed as 0 and 1, or as, good/bad, success/failure, profit/loss, in/out, or presence/absence of a particular characteristic.

Data having two classes is termed "binomial" (binomial = "two names") data. A measure can be inherently binomial, e.g. where a profit or loss is made, or of someone is in or out. Sometimes it is arrived at indirectly by categorizing some other numerical measure. Take, for instance, a case where telephone calls are classified on whether or not they last more than 10 min or, perhaps, whether or not they are answered within six rings.

The binomial distribution is a statistical distribution giving the probability of obtaining a specified number of particular outcomes (e.g. successes) in a specified number of independent trials with a constant probability of success in each.

The conditions that need to be satisfied for a binomial distribution are:

a) there is a fixed number of trials, n;

b) only two possible outcomes are possible at each trial;

c) the trials are independent;

d) there is a constant probability of a particular outcome e.g. success, "p", in each trial;

e) the variable is the total number of successes in n trials.

$$\text{mean} = np \quad (1)$$

$$\text{standard deviation} = \sqrt{np(1-p)} \quad (2)$$

The binomial distribution is very cumbersome to calculate. When dealing with control limits other distributions can be used as approximations to the binomial. For example, if p is small, in Equation 2, $(1 - p)$ approaches 1 and the standard deviation of the binomial distribution approximates to the square root of its mean. Equations (1) and (2) then become identical to equations (3) and (4) of the Poisson distribution.

In standard statistical process control methodology, binomial data is monitored using:

i) "p" charts for proportions, particularly when the sample size is variable; and

ii) "np" charts for numbers from samples of constant size.

4.3 Count data and the Poisson distribution

Count data relates to counts of events where each item of data is the count of the number of particular events per given period of time or quantity of product. Instances are: number of accidents or absentees per month, number of operations or sorties per day, number of incoming telephone calls per minute, or number of non-conformities per unit or batch.

The Poisson distribution has two principal parts to play in this standard:

i) as an approximation to the more cumbersome binomial when:

n is large and p is small, say, $n > 20$ and $p \leq 0.1$ (i.e. $\leq$ 10 %);

ii) as a distribution in its own right. The binomial distribution applies to the situation where an item, say, is classified as conforming or non-conforming. Here one can count either the number of items non-conforming or the number of items conforming. Poisson data arises when one can count only, say, the number of faults in an item not the number of non-faults. The conditions that need to be satisfied for a Poisson distribution are that events occur randomly, singly, uniformly and independently.

The validity of the Poisson model hinges on the independence of events and their occurrence at an average rate that is assumed to be stable (in the absence of special causes).

Two fundamental features of the Poisson distribution are:

a) the connection between its standard deviation and mean; and

b) the results of summation of two independent Poisson variables.

Regarding a), the mean and standard deviation of a Poisson distribution are given by:

$$\text{mean} = m \quad (3)$$

$$\text{standard deviation} = \sqrt{m} \quad (4)$$

This relationship is useful for establishing the validity of the Poisson model for a particular set of data. Whilst many real life situations can suggest that Poisson is a reasonable underlying model it would be unwise to take this for granted. Sometimes a degree of clustering of events is present, for example, of accidents and faults. This has the effect of increasing the spread beyond that of the Poisson distribution. This wider spread, termed over-dispersion, can gives rise to a higher false alarm rate when monitoring using control charts.

To reduce the risk of assuming an incorrect model a simple "dispersion" test is often recommended (see Annex A). This involves comparing the sample variance of the data with the mean.

Equation (4) indicates that the variance (the square of the standard variation) is equal to the mean for a Poisson distribution. Hence if:

a) sample variance equals the mean, the Poisson model is plausible;

b) sample variance is "much greater or smaller than" the mean, the Poisson model is implausible.

This variance comparison can be supported by other methods such as a graphical evaluation of the data and by goodness of fit tests such as the chi-squared test for differences between expected and observed frequencies.

Regarding b), distribution of the sum of independent Poisson variables is still Poisson. For example:

"if two sets of Poisson data are independent with separate means m_1 and m_2, their sum is also Poisson with mean, $(m_1 + m_2)$".

This fact has considerable impact on counts where multiple characteristics are involved. Instances are adding together different types of flaws, absenteeism for different reasons and injuries of different kinds. Another application is where events of a particular kind have a very small mean so that counts are frequently zero. This enables one to increase the number of sampled items (e.g. counts per month rather than per day or week).

In standard statistical process control, Poisson data is monitored using:

1) "*c*" charts for numbers (counts of events) from samples of constant size; and

2) "*u*" charts for proportions (number of events per sample), particularly when the sample size is variable.

5 Limitations of standard control chart limits: appropriate counter-measures

5.1 Bases for selection of control chart limits for count data

5.1.1 *Introduction*

It is common practice throughout the world to base standard control chart limits on the assumption of normality of data. Control limits are then calculated thus:

a) action control limits = mean ± 3 standard deviations, of the plotted statistic;

b) warning control limits = mean ± 2 standard deviations, of the plotted statistic.

In terms of count data using a "*c*" chart, these equations become:

$$\text{action control limits} = \bar{c} \pm 3\sqrt{\bar{c}} \tag{3}$$

$$\text{warning control limits} = \bar{c} \pm 2\sqrt{\bar{c}} \tag{4}$$

However, it should be appreciated that the "normal" distribution is symmetrical and bell shaped whereas the actual "Poisson" distribution, representative of count data, is frequently not so.

Does this really matter?

There are those who would argue that it doesn't really matter very much, as control chart limits are based on quite arbitrary considerations anyway. Anyhow, even if it does matter somewhat, taking it into account would make things more difficult and is likely to put a lot of people off using a very simple and useful tool.

Others would argue that it is purely a matter of the method of establishing control limits and does not add any complexity to the application of a control chart by the users. To ensure the validity, credibility and maximum effectiveness of control charts, there is a need to know both the consequences of the choice of control limits, and the appropriate countermeasures that can be taken, by people technically responsible for them.

This standard takes the view that people technically responsible for control charts need to be aware of the consequences of, and the measures required to correct for, any errors arising from the use of standard control chart limits for count data. This enables judgements to be made on the proper course of action to take in particular circumstances.

5.1.2 *Consequences*

Figure 1 compares the shape of the "Poisson" distribution for various values of the data mean.

The figure shows that standard conventional control limits, based on symmetry about the mean, are sometimes not a good reflection of the actual situation. Both the upper and lower control limits are adversely affected. The consequences of this are manifold. With respect to the application of the appropriate Poisson distribution, the symmetrical distribution used for standard conventional control chart limits, the most generally used, gives:

a) tighter upper control limits, progressively so with lower values of the mean;

b) slacker lower control limits, progressively so with lower values of the mean;

c) impossible negative values for low means.

An example of a situation where standard limits are not effective is the calibration of a particular meter that is prone to "sticking" to give the occasional zero reading. It is looked upon, erroneously, as "in control" in terms of the standard lower control limit, as zero is assumed because it calculates as negative with the low values of the mean experienced. Use of the Poisson-based control limit method provides correct lower control limits to indicate an "out of control" situation.

These differences in Poisson distribution patterns affect the risk probabilities associated with control limits at the mean ±2 standard deviations. When the distribution is not near normal, as is frequently the case with attribute data, these probabilities will vary from those calculated using standard conventional practice based on normality of data. For different values of process mean the risk associated with a decision concerning control of the process will change.

Constancy of risk, at the level obtained with standard Shewhart-type control limits for measured data, can usually readily be achieved using Poisson-based control limits rather than the conventional limits more generally used. These Poisson-based control limits are set out for various means in Annex B.

Figure 1 shows that for:

a) mean = 1, the pattern is decidedly non-symmetrical;

b) mean = 5, there is a tendency towards skewness to the right;

c) mean = 10, the skewness is less pronounced than at a mean = 5;

d) mean = 20, a near symmetrical normal pattern emerges.

The variation in the value of control chart limits between the conventional and Poisson methods of calculation (see Annex B) is shown in Table 1.

Table 1 confirms that the use of conventional control limits, rather than the more appropriate Poisson limits, can give rise to a fair amount of error and, in fact, in some cases, give impossible negative values.

Table 1 — Effect of method of calculation of action control limits in terms of value of the mean

Type of control limit	Mean = $\bar{c}$	Control limit values	
		Conventional $\bar{c} \pm 3\sqrt{\bar{c}}$	Poisson
Upper action	1	4.0	5.7
	5	11.7	13.7
	10	19.5	21.7
	20	33.4	35.7
Lower action	1	negative	0
	5	negative	0
	10	0.5	1.3
	20	6.6	7.3
NOTE Conventional action control limits are based on the standard formula $\bar{c} \pm 3\sqrt{\bar{c}}$			

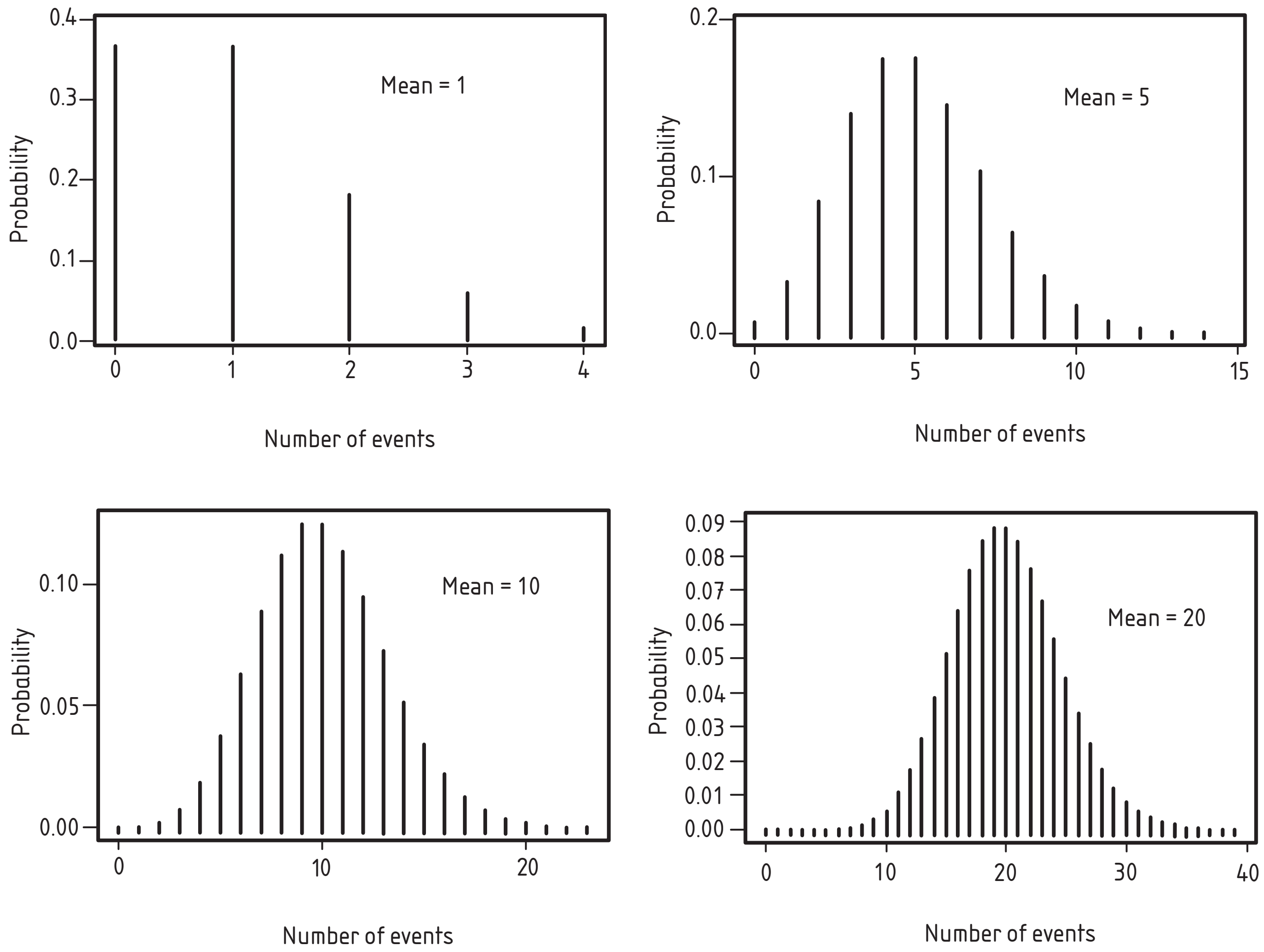

Figure 1 — Illustration of the variation in distribution shape with changes in the mean

5.1.3 ***Countermeasures***

The rational approach to the setting of control limits for count data is indicated in the flow chart of Figure 2.

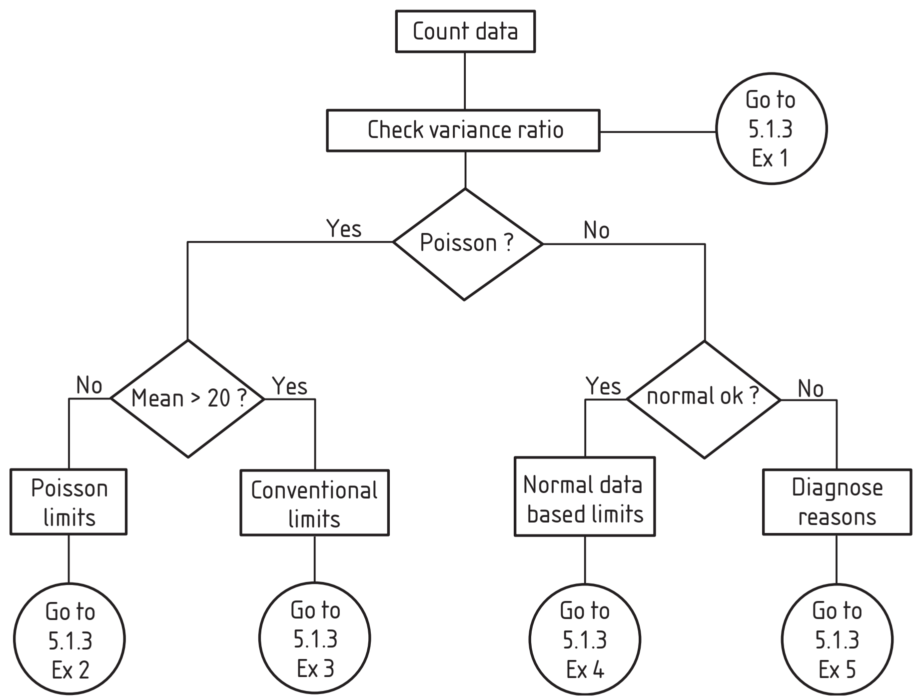

Figure 2 — Count data control limit selection flow chart

The flow chart of Figure 2 is used to construct data-based control chart limits for count data. Stages are as follows.

1) Apply Poisson variance ratio test to the count data. Use Annex A: Steps 1 to 3. (See Example 1.)

2) If the test indicates that the data:

a) appears to confirm a Poisson distribution go to Stage 3;

b) if not go to Stage 5.

3) If the data mean is less than 20, select Poisson-based control limits using Annex B (see Example 2) as conventional control limits give rise to a margin of error.

4) If the data mean is of the order of 20 or more, an appropriate approximation is to select conventional control limits using standard formulae. (See Example 3.)

5) Compare the data distribution with the normal distribution.

a) If the comparison is good select normal data-based control limits, using the observed rather than theoretical Poisson-based standard deviation. It is advisable also to investigate the possible causes for the unexpected non-Poisson type distribution using the methods of 5b). (See example 4.)

b) If not, diagnose the reasons for abnormality using Annex A: Steps 4 and 5. (See Example 5.)

EXAMPLE 1 Poisson variance ratio test

Project

The number of accidents, per four-week period, in a particular organization, are shown for 25 consecutive time periods.

12 5 7 10 9 5 11 6 7 9 5 11 9 4 12 6 11 9 7 4 11 10 8 7 6

Apply the Poisson variance ratio test as shown in Annex A.

Solution

Observed mean of data = $\bar{X} = \frac{1}{g}\sum X = 8.04$ (or use a calculator) (where g = number of samples)

Observed variance of data = $\frac{1}{g-1}\left[\sum(X-\bar{X})^2\right] = 6.46$ (or use a calculator and square the readout from the σ_{n-1} key).

Estimated theoretical variance of data (based on Poisson distribution) = sample mean = 8.04

$$V = \frac{\text{observed variance}}{\text{estimated theoretical variance}} = \frac{S^2}{\bar{X}} = \frac{6.46}{8.04} = 0.80$$

From Table A.1, for number of samples = g = 25, critical values are: 0.41 to 1.90 at the 1 % level of significance. V = 0.80 is within this range therefore there is no reason to suppose the distribution of the data is other than Poisson.

EXAMPLE 2 Poisson-based control limits

Project

Establish a control chart, with action limits, for the data of Example 1.

Solution

The solution to Example 1 indicates that there is no reason to suppose the distribution of the data is other than Poisson. A run chart of the data indicates that the data appears reasonably stable.

The mean = 8.04. This value is less than 20 therefore the Poisson distribution is used to calculate control limits. Reading from the tables in Annex B, the appropriate Poisson control limits are as shown in Table 2. The corresponding control limits for the inappropriate conventional method are also given for comparison to indicate the degree of error involved.

Table 2 — Example of difference between conventional and Poisson-based control limits

Type of control limit	Control limit values for mean = 8.04	
	Poisson-based (appropriate)	Conventional (appropriate)[a]
Upper action	18.7	16.5
Lower action	0.3	Negative
Upper warning	14.7	13.7
Lower warning	2.3	2.4

[a] calculated from the conventional formulae:
action control limits = mean $\pm 3\sqrt{\text{mean}}$, of the plotted statistic = $\bar{c} \pm 3\sqrt{\bar{c}}$
warning control limits = mean $\pm 2\sqrt{\text{mean}}$, of the plotted statistic = $\bar{c} \pm 2\sqrt{\bar{c}}$

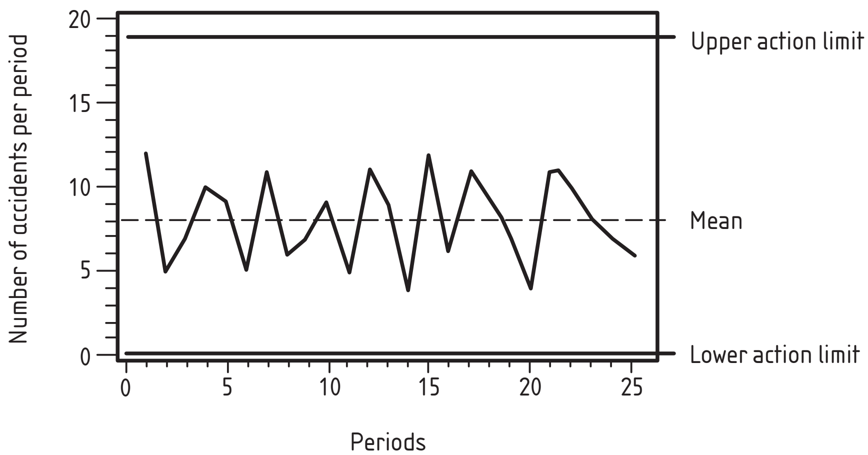

Figure 3 — "c" chart for accidents

EXAMPLE 3 Conventional Shewhart-type control limits

Project

The number of orders completed per day over 30 consecutive working days are as shown:

23 19 20 22 21 27 16 35 25 21 17 28 17 22 14 28 32 26 20 25 19 20 24 22 26 23 19 27 28 29

Establish a "*c*" control chart for this data.

Solution

First it is necessary to apply the Poisson variance ratio test as described in Annex A.

observed mean = 23.17
observed variance = 23.52
estimated theoretical variance = 23.17

$$V = \frac{23.52}{23.17} = 1.02$$

From Table A.1, for g = 30 critical values are 0.55 to 1.58 at the 1 % level of significance. With V = 1.02, the data is thus taken to reflect a Poisson distribution with a mean greater than 20. Hence it can be considered convenient to select conventional control limits, as an approximation, using the standard symmetrical formulae:

action control limits $= \text{mean} \pm 3\sqrt{\text{mean}}$, of the plotted statistic

$$= \bar{c} \pm 3\sqrt{\bar{c}}$$

$$= 23.17 \pm 3\sqrt{23.17}$$

$$= 23.17 \pm 14.44$$

warning control limits $= \text{mean} \pm 2\sqrt{\text{mean}}$, of the plotted statistic

$$= \bar{c} \pm 2\sqrt{\bar{c}}$$

$$= 23.17 \pm 2\sqrt{23.17}$$

$$= 23.17 \pm 9.63$$

These conventional control chart limits are shown in Table 3. The corresponding Poisson control limits are shown for comparison purposes to indicate the degree of difference involved with means of just over 20.

Table 3 — Conventional v Poisson control limits

Type of control limit	Control limit value	
	conventional	Poisson-based
Upper action	37.6	39.7
Lower action	8.7	9.3
Upper warning	32.8	33.7
Lower warning	13.5	13.3

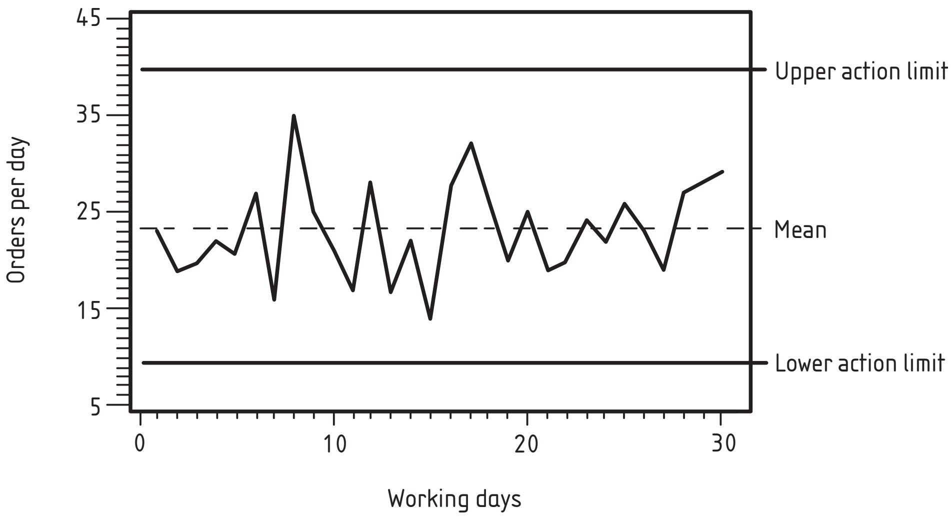

Figure 4 — "*c*" chart for orders per day

EXAMPLE 4 Normal distribution data based control limits

Project

The number of adjustments to test parameters required per complex unit on functional test, over a 50-unit period, is shown in assembly sequence, thus:

14 16 18 17 21 14 14 17 15 18 12 17 17 15 15 17 18 14 12 19 15 16 19 15 20

14 13 11 16 16 14 16 17 13 15 18 19 12 17 18 15 19 13 16 15 13 16 16 18 20

Set up an appropriate control chart.

Solution

First, it is necessary to apply the Poisson variance ratio test as described in Annex A.

$$\text{Observed mean of data} = \bar{X} = \frac{1}{g}\sum X = 15.90$$

$$\text{Observed variance of data} = \frac{1}{g-1}\left[\sum (X-\bar{X})^2\right] = 5.40$$

Estimated theoretical variance of data (based on Poisson distribution) = sample mean = 15.90.

$$V = \frac{\text{observed variance}}{\text{estimated theoretical variance}} = \frac{S^2}{\bar{X}} = \frac{5.40}{15.90} = 0.34$$

From Table A.1, for number of samples = g = 50, critical values are: 0.56 to 1.60 at the 1 % level of significance. V = 0.34 is outside this range therefore it is not appropriate to suppose the distribution of the data is Poisson. **A.2** suggests that this can arise due to some regular or systematic feature of the process rather their random or independent occurrences.

To establish control chart limits for this situation, it is necessary, initially, to check for normality of the data. This can be achieved in a number of ways. The following graphical methods are simple to apply and are also very effective visually.

a) Plot a histogram of the data and compare visually with the symmetrical bell shaped, normal distribution.

b) Plot the cumulative data on a normal probability worksheet. A straight line that indicates the data are normal.

The histogram is shown in Figure 5 and the normal probability plot in Figure 6. Both indicate that the data are normal-like in form.

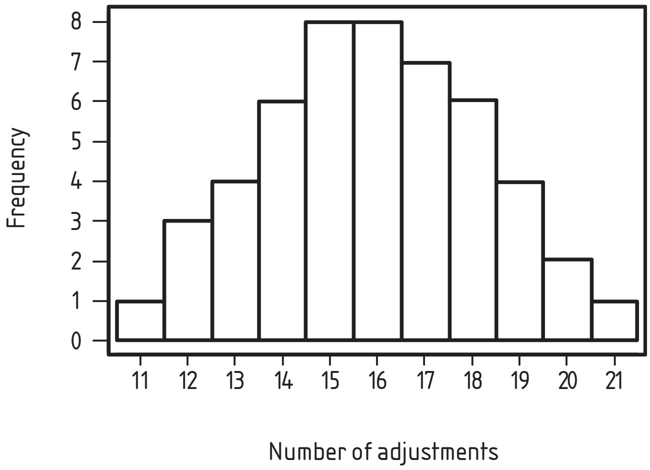

Figure 5 — Histogram of adjustment data

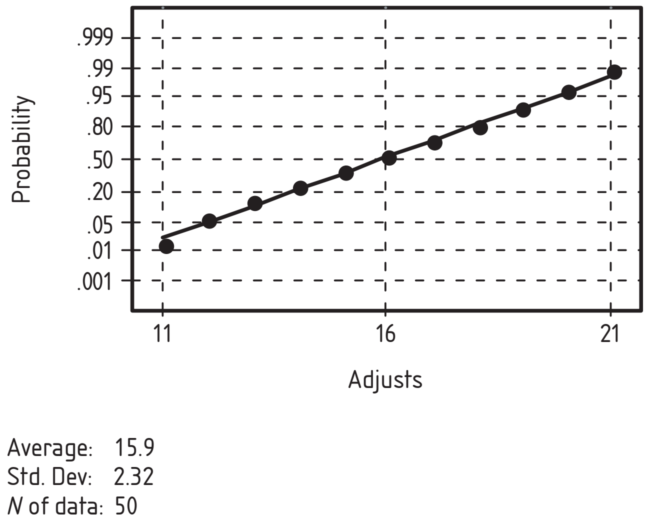

Figure 6 — Normal probability plot of adjustment data

Control limits can now be established using the formulae:

Action limits = mean ±3 (observed standard deviation)

Warning limits = mean ±2 (observed standard deviation)

Note the observed standard deviation, $S = \sqrt{\frac{1}{g-1}\left[\sum(X-\bar{X})^2\right]}$ is used here and not the estimated theoretical standard deviation based on the Poisson distribution. This standard deviation can readily be determined using an appropriate calculator. Use the σ_{n-1} key, not the σn one.

Thence:

Action limits = 15.9 ± (3 × 2.32) = 15.9 ± 7.0

Warning limits = 15.90 ± (2 × 2.32) = 15.9 ± 4.6

However, as the standard deviation of 2.32 is so much smaller than that expected (namely $\sqrt{15.9} \approx 4$) there is a need to investigate the reasons for this. There appears to be some possibility of consistent suppression of extreme values throughout. The consequent control chart for number of adjustments per unit is shown in Figure 7.

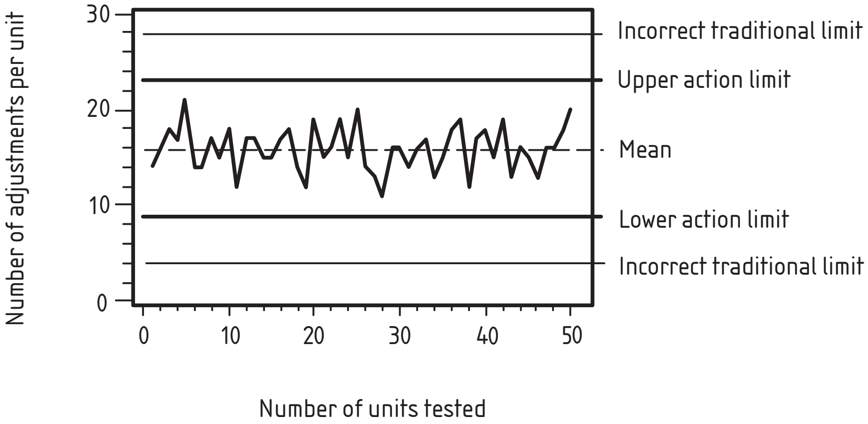

Figure 7 — Control chart for number of adjustments per unit

EXAMPLE 5 Peculiar data diagnosis

Project

Faults per 100 m roll of fabric are recorded. To date the events of faults per roll, in production sequence, are:

7 5 8 3 1 4 6 3 3 6 4 7 5 2 4 2 2 4 1 2 5 2 0 3 5 1 3 2 4 1

2 1 0 2 1 3 4 0 2 1 3 1 0 3 1 0 2 1 0 1 3 0 1 2 1 0 0 2 1 0

It is required to establish an appropriate control chart.

Solution

First it is necessary to apply the Poisson variance ratio test as described in Annex A.

$$\text{Observed mean of data} = \bar{X} = \frac{1}{g}\sum X = 2.38$$

$$\text{Observed variance of data} = \frac{1}{g-1}\left[\sum (X-\bar{X})^2\right] = 3.94$$

Theoretical variance of data (based on Poisson distribution) = mean = 2.38

$$V = \frac{\text{observed variance}}{\text{estimated theoretical variance}} = \frac{S^2}{\bar{X}} = \frac{3.94}{2.38} = 1.65$$

From Table A.1, for number of samples = g = 60, critical values are: 0.59 to 1.54 at the 1 % level of significance. V = 1.65 is above this range therefore it is not appropriate to suppose the distribution of the data is Poisson.

The histogram of Figure 8, too, indicates that the data is decidedly non-normal.

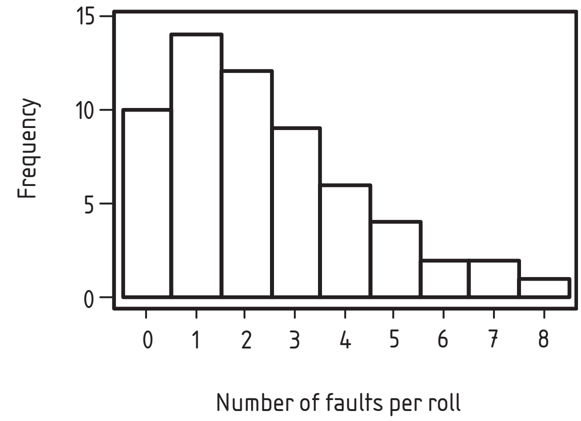

Figure 8 — Histogram of fabric fault data

Subclause **A.2**, Step 4, suggests that the large value of variance ratio here can arise due to changes in the average rate of events.

The next stage of diagnosis is thus to plot a run chart of number of faults per roll in terms of production sequence. The run chart is shown in Figure 9.

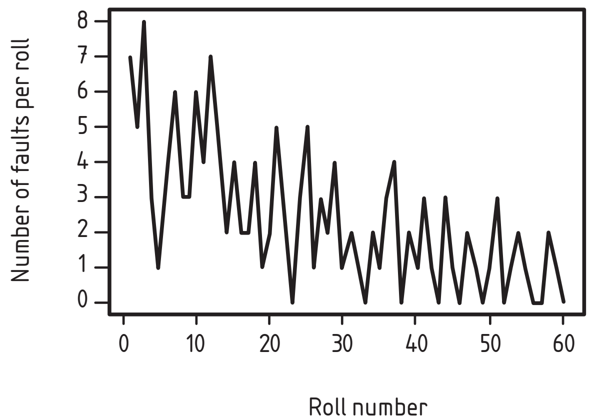

Figure 9 — Run chart of fabric faults indicating significant improvement in performance

Observation of the run chart (confirmed by cusum analysis; see BS 5703) indicates two distinct stages of fault performance improvement thus:

a) at the 13th roll the process mean dropped from about 4.75 faults per roll to about 2.44 faults per roll;

b) at the 38th roll the process mean dropped further from about 2.44 to about 1.09 faults per roll.

These improvements related to the results achieved from a two-stage quality improvement project.

Control chart limits should reflect current performance. From roll 38 to 60, the observed standard deviation is 1.04. This coincides precisely with the theoretical value of $\sqrt{1.09} = 1.04$. Hence, using the tables in Annex B for mean = 1.09, the control chart limits for current performance are set at:

i) upper action limit = 5.7;

ii) upper warning limit = 4.7.

The marked-up run chart together with the current control limits are shown in Figure 10.

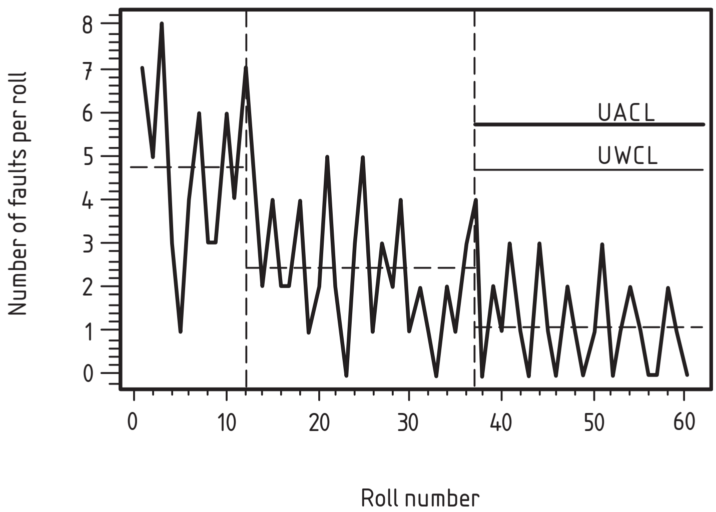

Figure 10 — Control chart for number of faults per roll

5.1.4 *Handling variations in sample size*

When there is variation in sample sizes, the "*u*" chart, as described in BS 5701-2, is used. The same limitations and countermeasures as those discussed for the "*c*" chart apply. As suggested in BS 5701-2, it is better to avoid variations in sample size if practicable.

5.2 Bases for selection of control limits for classified data

5.2.1 *Introduction*

It is common practice throughout the world to base standard control limits for classified data on the assumption of normality of data. Control limits are then calculated thus:

a) action control limits at: mean ±3 standard deviations;

b) warning control limits at: mean ±2 standard deviations.

These equations are identical to those used for measured data control limits.

In terms of classified data using an "*np*" chart, these expressions become:

action control limits = $n\overline{p} \pm 3\sqrt{n\overline{p}(1-\overline{p})}$

warning control limits = $n\overline{p} \pm 2\sqrt{n\overline{p}(1-\overline{p})}$

However, it should be appreciated that the "normal" distribution is symmetrical and bell shaped, whereas the actual "binomial" distribution, representative of classified data, is frequently not so.

Does this really matter? Similar reasoning applies as with count data already discussed (see **5.1.1**).

This standard takes the view that people technically responsible for control charts need to be aware of the consequences of, and the measures required to correct for, any errors arising from the use of standard control chart limits for count data. This enables judgements to be made on the proper course of action to take in particular circumstances.

5.2.2 ***Consequences***

Figure 11 compares the shape of the "binomial" for various values of the data mean. As with the Poisson distribution, the binomial is far from symmetrical at lower values of the mean (*np*) and, similarly, gets progressively more symmetrical with increases in the mean.

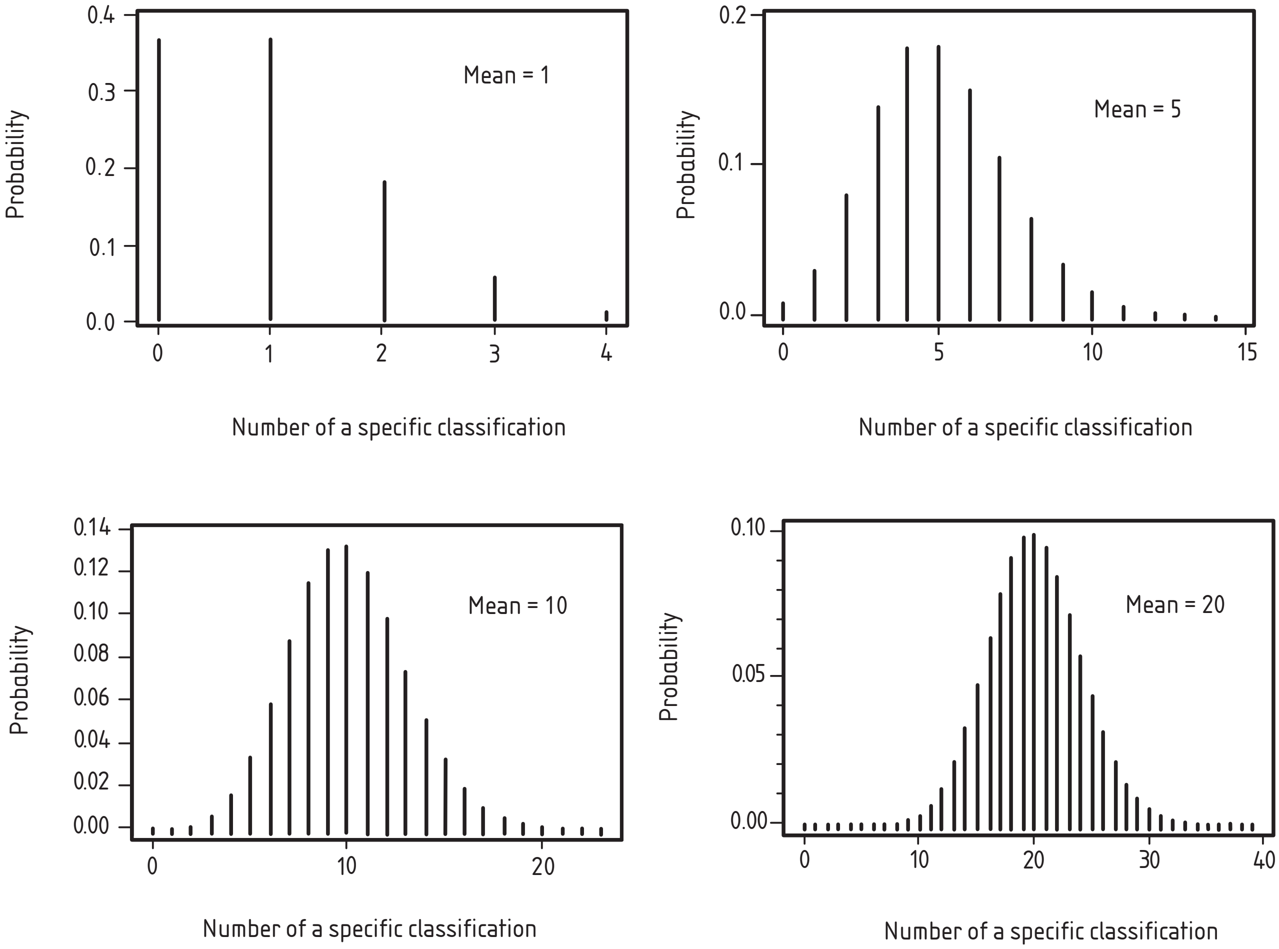

Figure 11 — Change in shape of the binomial distribution with different means

The consequences are similar to that discussed in **5.1.2**. Differences are particularly marked at the lower control limit. Two main conclusions are drawn in this respect:

a) the lower the value of "*np*", the less the value of the traditional lower control limit in signalling significant reductions in "*p*";

b) the lower control limit is indicated as negative (an impossibility) if $n < \frac{9(1-\bar{p})}{\bar{p}}$. Hence it is often advantageous to select a sample size, "*n*", to give a lower control limit greater than zero.

The principal difference is that, with count data, Poisson-based control limit values are readily expressed in tabular form, being based purely on the mean. With classified data, a complication arises as binomial-based control limit values depend on both "*p*" and the sample size or area of opportunity, *n*, so they become more tedious to calculate manually. Consequently, binomial tables need extensive listings both in terms of "*n*" and "*p*".

5.2.3 ***Countermeasures***

The rational approach to the setting of control limits for classified data is indicated in the flow chart of Figure 12.

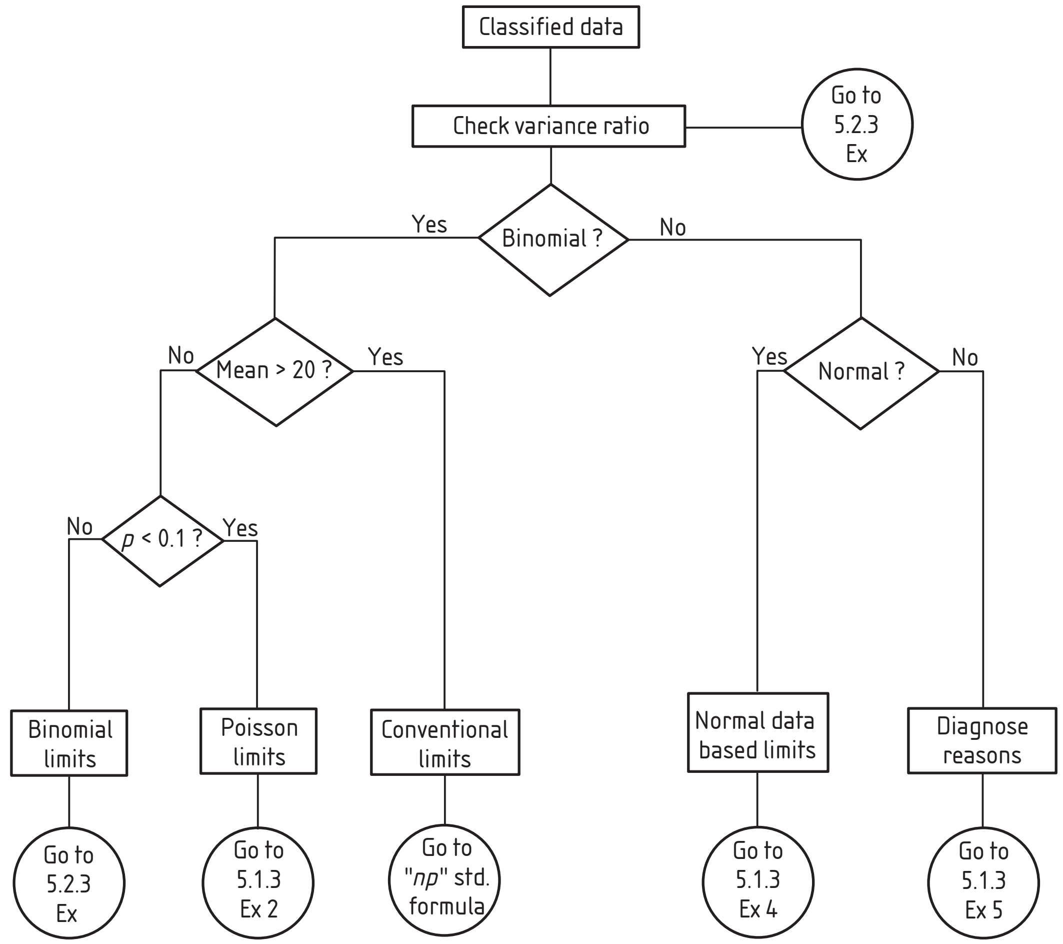

Figure 12 — Classified data distribution selection flow chart

The flow chart of Figure 12 is used to construct data based control chart limits for classified data. Stages are as follows.

a) Apply binomial variance ratio test to the classified data. Use Annex A.

b) If the test indicates that the data:

1) appear to reflect a binomial distribution, go to Annex A, Stages 3 or 4,

2) if not go to Stage 5.

c) If the data mean, (np), is less than 20 and:

1) p is greater than 0.1 (10 %), use binomial control limits;

2) p is less than or equal to 0.1, an appropriate approximation is Poisson control limits.

d) If the data mean is 20 or more, an appropriate approximation is to select conventional limits using standard formulae.

e) Compare the data distribution with the normal distribution:

1) if the comparison is good, select normal data based control limits. Use the observed rather than the theoretical binomial based standard deviation. It is also advisable to investigate the possible causes for the unexpected non-binomial type distribution using the methods of **5.2.3**e)2);

2) if the comparison is not good, diagnose the reasons for abnormality using Annex A; Steps 4 and 5.

An example is shown illustrating how to handle the binomial case of **5.2.3**c)1). All other situations are identical to those covered in Examples 2 to 5 of **5.1.3** except that the variance ratio test in each case relates to the binomial rather than the Poisson distribution.

EXAMPLE

Project

It is required to set up a control chart with action limits to monitor non-conforming spot welds on a particular fabricated housing. There are 50 spot welds per housing. Housings are fabricated in batches of 10. Results taken of the number of non-conforming welds per batch on 25 batches in production sequence are:

8, 6, 9, 5, 8, 15, 14, 16, 9, 14, 11, 8, 11, 13, 8, 10, 8, 4, 13, 10, 6, 11, 9, 12, 9

Solution

Stage 1: Apply binomial variance ratio test to data

Estimate:

$$\text{Observed mean} = n\bar{p} = \frac{\text{sum of np's}}{\text{number of np's}} = \frac{247}{25} = 9.88$$

$$\text{Observed variance} = \frac{1}{g-1}\left[\sum(np - n\bar{p})^2\right] = 9.78$$

where

g = number of samples = 25

n = number in a sample = number of welds per housing × number of batches in each sample = 50 × 10 = 500

p = probability of a non-conformance in any individual weld = $\frac{9.88}{500} = 0.019\ 8$

$$\text{Estimated theoretical variance} = np(1-p) = 9.88(1-0.019\ 8) = 9.68$$

$$V = \frac{\text{observed variance}}{\text{estimated theoretical variance}} = \frac{9.78}{9.68} = 1.01$$

Stage 2

From Table A.1, critical values of V are 0.45 to 1.81. V, at 1.01, is well within these critical values, hence there is no reason to suppose that the data is other than a binomial distribution.

Stage 3:

The data mean, $n\bar{p}$, is less than 20 and p is less than 0.1, therefore control limits can be based on the Poisson approximation. Using the tables in Annex B for a Poisson mean of 9.88:

UCL = 21.7;

LCL = 1.3

A control chart can now be established as shown in Figure 13.

To indicate the relative simplicity of using the Poisson approximation to the binomial, the control limits are calculated using the binomial distribution.

Refer to binomial tables or use a computer program to determine the binomial control chart limits.

Binomial tables list probabilities of obtaining x or fewer particular outcomes in a single trial.

For n = 500 and p = 0.019 8, results are shown in Table 4.

Table 4 — Binomial probabilities for n = 500: p = 0.019 8

x	0	1	2	3	4	5	6	7	8
probability of x or less	0.000 05	0.000 50	0.002 82	0.010 57	0.030 03	0.069 03	0.134 03	0.226 67	0.342 01
x	9	10	11	12	13	14	15	16	17
probability of x or less	0.469 36	0.595 68	0.709 34	0.802 90	0.873 84	0.923 69	0.956 32	0.976 29	0.987 78
x	18	19	20	21	22	23	24	25	26
probability of x or less	0.994 01	0.997 20	0.998 75	0.999 47	0.999 78	0.999 92	0.999 97	0.999 99	1.000 00

In this example:

With reference to the upper control limit set at 0.001 35 probability.

From Table 4:

Probability of 19 or fewer = 0.997 20
∴ Probability of 20 or more = 1 – probability of 19 or less = 0.002 80 (in control as >0.001 35)
Probability of 20 or less = 0.998 75
∴ Probability of 21 or more = 0.001 25 (out of control as less than 0.001 35)
∴ Upper control limit (UCL) = 21 → 20.7 (to make it quite clear visually that 21 is "out of control")

With reference to the lower control limit set at 0.001 35 probability

From Table 4:

Probability of 2 or less = 0.002 82 (in control as more than 0.001 35
Probability of 1 or less = 0.000 50 (out of control as less than 0.001 35)
∴ Lower control limit (LCL) = 1 → 1.3 (to make it quite clear visually that 1 is "out of control").

A comparison of the two sets of control limits calculated using the binomial and the Poisson approximation indicates that, in this case:

a) the upper control limit is marginally higher in the Poisson case. The upper control limits would have been identical if the mean had been 9.86 rather than 9.88,

b) the lower control limits are identical.

It should be appreciated that this "*np*" control chart does not pinpoint:

a) housing to housing variation within a batch;

b) weld to weld variation within a housing.

These sources of variation need to be exploited in diagnostic activities aimed at:

i) locating and removing the source of any special cause variation;

ii) improving the performance capability of a weld by reducing common cause variation.

5.2.4 *Handling variations in sample size*

When there is variation in sample sizes the "*p*" chart as described in BS 5701-2 is used. The same limitations and countermeasures as that discussed for the "*np*" chart apply. As suggested in BS 5701-2 it is better to avoid variations in sample size if at all possible and feasible.

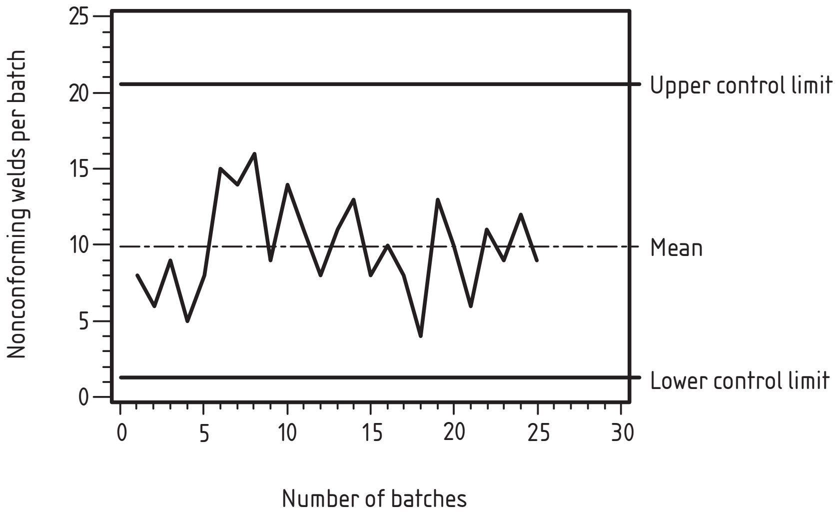

Figure 13 — Control chart for non-conforming weld data

6 Specialized design control charts

6.1 Universal attributes charts (*c*, *u*, *np* and *p*)

6.1.1 *Purpose*

The universal attribute chart is used for continuity purposes, as it permits different items, having dissimilar performance levels, to be plotted on an attribute statistical process control (SPC) chart.

6.1.2 *Method*

Transform data before plotting and using for SPC, as shown in Table 5.

Table 5 — Plotting data for standardized control charts

Chart	Parameter			
	Plot point	Centre line	UCL	LCL
c	$\frac{c - \bar{c}_{\text{target}}}{\sqrt{\bar{c}_{\text{target}}}}$	0	+3	–3
u	$\frac{u - \bar{u}_{\text{target}}}{\sqrt{\frac{\bar{u}_{\text{target}}}{n}}}$	0	+3	–3
np	$\frac{np - n\bar{p}_{\text{target}}}{\sqrt{n\bar{p}_{\text{target}}\left(1 - \frac{n\bar{p}_{\text{target}}}{n}\right)}}$	0	+3	–3
p	$\frac{p - \bar{p}_{\text{target}}}{\sqrt{\frac{\bar{p}_{\text{target}}\left(1 - \bar{p}_{\text{target}}\right)}{n}}}$	0	+3	–3

6.1.3 *Example*

Threads are being checked at a particular workstation for visual non-conformities. Two types of stud are currently being run, X and Y. "Just In Time" production is being applied, so batch and sample size can vary. Each bolt has a different level of expected quality performance, namely target $\bar{u}$, as shown.

Calculate the plotting points for a new stud, stud Z, and analyse the consequent control chart for out of control conditions.

Table 6 — Example of tabulation for setting up a universal "*u*" chart

Item	X	X	Y	Y	Y	X	Y	X	X	X	Z	Z	Z	Z	Z
Sample size (n)	20	25	40	50	40	25	40	20	25	25	100	80	100	90	100
Number (c)	2	3	2	2	1	0	0	1	3	2	24	16	14	23	36
Proportion (u)	0.10	0.12	0.05	0.04	0.025	0	0	0.05	0.12	0.08	0.24	0.20	0.14	0.255	0.36
Target ($\bar{u}$)	0.10	0.10	0.05	0.05	0.05	0.10	0.05	0.10	0.10	0.10	0.20	0.20	0.20	0.20	0.20
Plot point	0	0.32	0	–0.32	–0.71	–1.58	–1.41	–0.71	0.32	–0.32	0.89	0	–1.34	1.18	3.58

The plotting positions for stud Z are found thus:

With reference to the plot point 1 for Z.

$$c_1 = 24: n_1 = 100: u_1 = \frac{c_1}{n_1} = \frac{24}{100} = 0.24 \text{ target } \bar{u} = 0.20$$

$$\therefore \text{plot point}_1 = \frac{u_1 - \bar{u}_{\text{target}}}{\sqrt{\frac{\bar{u}_{\text{target}}}{n_1}}} = \frac{0.24 - 0.20}{\sqrt{\frac{0.20}{100}}} = +\frac{0.04}{0.044\ 7} = +0.89$$

With reference to the plot point 2 for Z.

$$c_2 = 16: n_2 = 80: u_2 = \frac{16}{80} = 0.20$$

$$\therefore \text{plot point}_2 = \frac{u_2 - \bar{u}_{\text{target}}}{\sqrt{\frac{\bar{u}_{\text{target}}}{n_2}}} = \frac{0.20 - 0.20}{0.05} = 0$$

With reference to the plot point 3 for Z.

$$c_3 = 14: n_3 = 100: u_3 = \frac{14}{100} = 0.14$$

$$\therefore \text{plot point}_3 = \frac{u_3 - \bar{u}_{\text{target}}}{\sqrt{\frac{\bar{u}_{\text{target}}}{n_3}}} = \frac{0.14 - 0.20}{0.044\ 7} = -1.34$$

Figure 14 indicates an out-of-control situation at the fifth plot point for stud Z. A special cause for this should be sought.

The formulae can appear rather complex using this approach. However, it does have the effect of simplifying the construction of the actual control chart in that all such charts have a centre-line at zero and control limits at ±3.

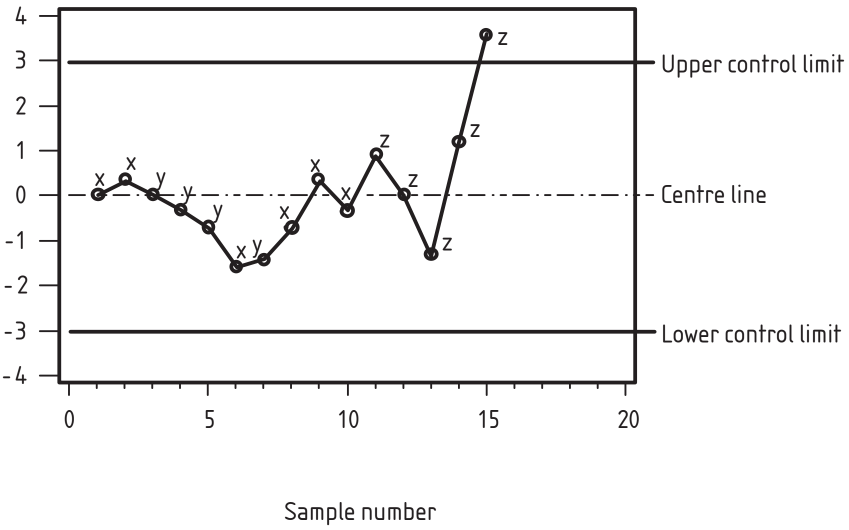

Figure 14 — Universal "*u*" chart for stud threads

6.2 Differing severity (weighted) characteristics chart

6.2.1 *Purpose*

This chart is useful when the characteristics (e.g. non-conformity) plotted have different levels of severity.

6.2.2 *Scope*

Conventional multiple characteristic control charts for non-conformities (standard "*c*" charts) assume that one type of non-conformity is no more or less severe in consequence than another.

This often does not reflect reality. Some faults:

— can be of a minor cosmetic nature (e.g. slight uneven stitching on the cuff of a shirt, or slight paint runs on an automotive oil filter cover located out of sight at the bottom of an engine);

— can be of major functional consequence (e.g. severe delamination of the collar of a shirt or pin holes in an oil filter).

To handle this situation from an SPC point of view, it is necessary to use a system of demerits. Here a minor blemish might carry a weighting (or penalty) of 1 demerit, a major fault 10 demerits and a critical one, 100 demerits. The relative weighting in a particular situation will depend on an assessment of features such as technical consequences and economics.

6.2.3 *Method*

Set up a multiple characteristic control chart to the criteria in Table 7.

Table 7 — Formulae for setting up a demerits attribute chart

Plot point	$D = \sum wc$
Centreline	$\overline{D} = \sum w\overline{c}$
Action control limits	CL's $= \overline{D} \pm 3\sqrt{\sum\left(w^2\overline{c}\right)}$
Warning control limits	CL's $= \overline{D} \pm 2\sqrt{\sum\left(w^2\overline{c}\right)}$

where

D = total number of demerits per sample (or sub-group)
c = number of non-conformities per sample of each demerit class
w = numerical values of weight or penalty of each demerit class

EXAMPLE

Five panels per hour are audited for non-conformities following stamping in a press area. Checking characteristics are weighted according to severity thus:

$w_1 = 1.0$: $w_2 = 0.5$: $w_3 = 0.1$

The results of 20 consecutive audits are shown in Table 8.

Table 8 — Results of twenty consecutive audits in multiple characteristic control chart format

Weight (w)	**Checking characteristic**	**Non-conformities** (c)																			
1.0	*Heavy scores*																				
	Splits			1			1								2						
	Holes missing																				
0.5	*Heavy burrs*																				
	Slug marks									1										1	
	Strains						1														
	Dings and dents							2													
0.1	*Dings and dents*	3	2	2	3								3	2				1	2	2	
	Slug marks																1				
	Buckles																1				
	Strains																				1
	Burrs																				
Demerits (D)		0.3	0.2	1.2	0.3	0	1.5	1.0	0	0.5	0	0	0.3	0.2	2.0	0	0.2	0.1	0.2	0.7	0.1
Sample size		<--------------------5-------------------->																			

The calculations involved are:

$$\bar{c}_1 = \frac{4}{20} = 0.2 \qquad \bar{c}_2 = \frac{5}{20} = 0.25 \qquad \bar{c}_3 = \frac{23}{20} = 1.15$$

where the subscript for each average non-conformity, $\bar{c}$, relates to the corresponding weight, w, subscript:

$$w_1 = 1.0 \qquad w_2 = 0.5 \qquad w_3 = 0.1$$

$$w_1\bar{c}_1 = 0.2 \qquad w_2\bar{c}_2 = 0.125 \qquad w_3\bar{c}_3 = 0.115$$

Centreline $= \overline{D} = \sum w\bar{c} = 0.2 + 0.125 + 0.115 = 0.44$

Control limits $= \overline{D} \pm 3\sqrt{\sum w^2\bar{c}}$

$$= 0.44 \pm 3\sqrt{(1 \times 0.2) + (0.25 \times 0.25) + (0.01 \times 1.15)}$$

7 Attribute charts for measured data situations

7.1 Overview

Attribute control charts are sometimes used in cases where measured data control charts could be applied. This course is taken in the interests of simplicity, cheapness or speed at the expense of some loss of sensitivity. This gives rise to yes/no, binary type decisions where a specified limit is involved. For example:

— a train is either late or not;

— telephone calls that last more or less than 10 min, say;

— a gauged item is labelled "go" or "no go";

— a unit tested for multiple measured data characteristics is signalled by either a red light (NOT OK) or a green one (OK).

Two situations arise:

a) low performance situation; the rate of occurrence of "out of specification" events is sufficiently high to be plotted directly on a standard or modified chart. This situation is dealt with in **7.2**;

b) high performance situation; low, or zero, rate of occurrence of "out of specification" events. This situation is dealt with in **7.3**.

Figure 15 shows a flow-chart illustrating the basis for selection of the appropriate control charts for measurable data.

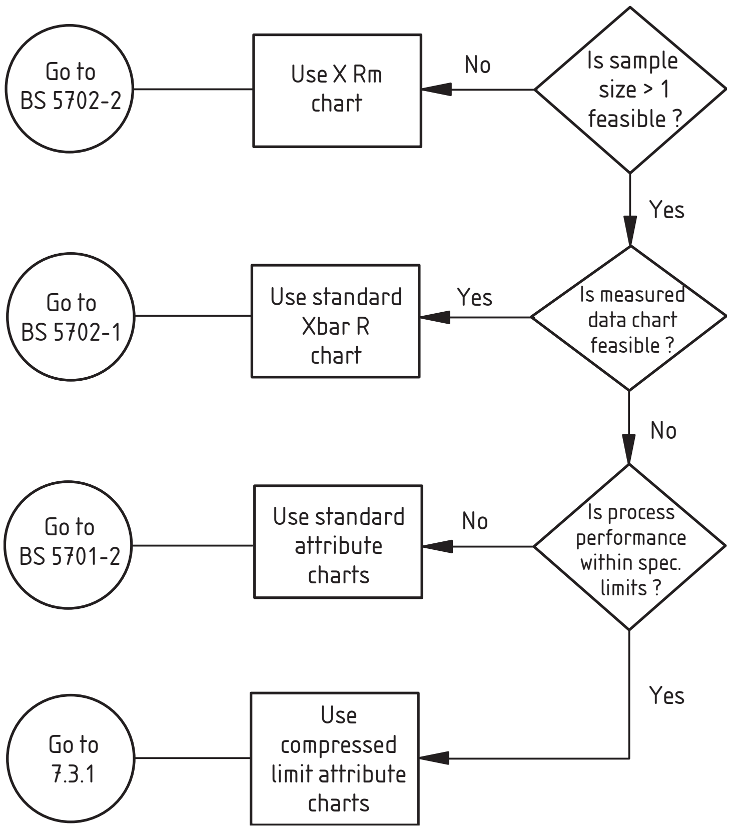

Figure 15 — How to select the appropriate control chart for measurable data

7.2 Low performance situation

7.2.1 *Introduction*

Two conditions are possible.

— Only one specification limit is involved, upper or lower. In this case, the standard methods described in BS 5701-2 are appropriate.

— Two limits, upper and lower, are involved. This involves the use of two-way control charts as described in **7.2.2**.

7.2.2 *Two-way control charts*

When examination is against upper and lower specification limits, two sets of data arise, events above the upper limit and events below the lower limit. Such data are best portrayed in a two-way attribute chart, as shown in Figure 16a). On one chart the number of events/items outside the upper limit are plotted in the normal way. On another, lower chart, the number of events/items outside the lower limit is plotted, in inverted form, on the same base as the upper chart. Often the two-way chart is supported by a further, third chart, on the same base. This chart represents the total number of events/items outside of both specification limits.

Figures such as Figure 16a) and Figure 16b) can be used to diagnose and assess performance in terms of:

a) a shift in the mean;

b) a change in variability;

c) out-of-control situation;

d) process capability.

A shift in the mean only will show as a decrease in the number of events at one limit and an increase to the other. This produces a very clear shift in the points plotted on the top two-way chart, giving a clear indication of change (see Figure 16b).

An increase in the variability alone will show a greater dispersion between the plotted points on the top two-way chart. A combination of change in mean and increase in the variability will exhibit features of both changes. These are clearly illustrated in Figure 16a) and Figure 16b).

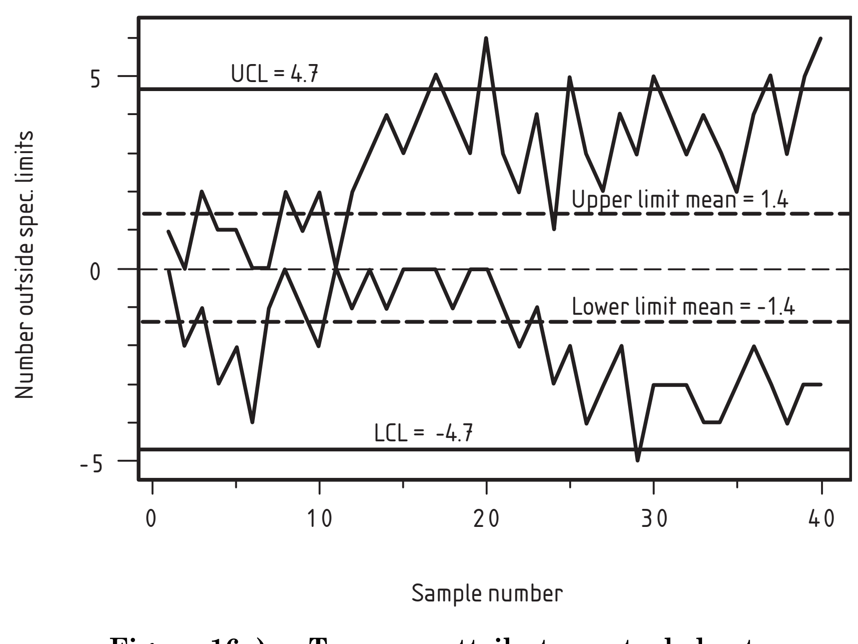

Figure 16a) — Two-way attribute control chart

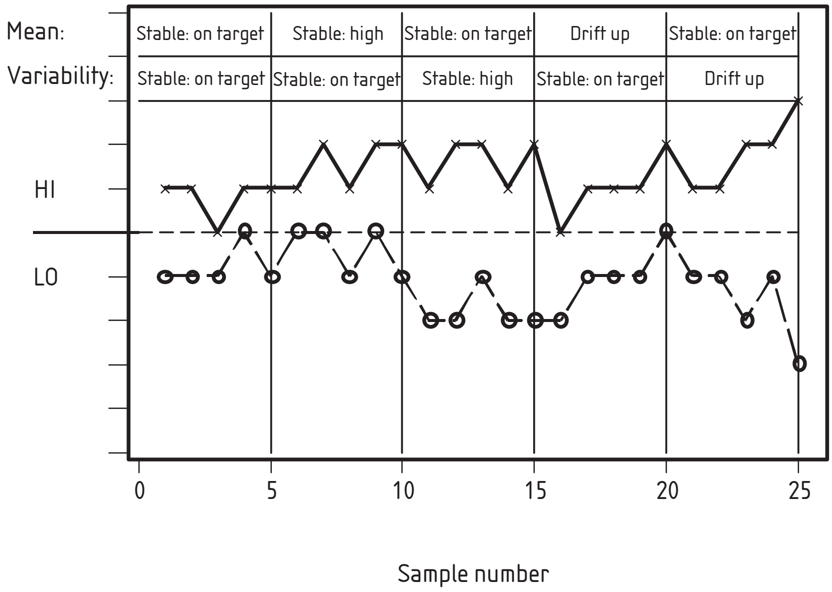

Figure 16b) — Illustration of process diagnosis using two-way control charts

7.3 High performance situation

7.3.1 *Introduction to compressed limit control charts*

It is well known that standard attribute control charts suffer from disadvantages that make them unsuitable when a zero, or a very low, proportion of events outside of specification exist or are tolerated. This disability arises because such charts can give signals only when one or more events to be plotted appear in a sample.

One possible course of action that should be investigated in such cases is to increase the sample size or to compound a series of samples and follow the method prescribed in **7.2**.

However, in very high performance processes the sample size can become prohibitively high, both on the grounds of economy of effort and/or time lag in reaching a decision. Of course, if no "out of specification" events are permissible, then the method is not feasible.

A compressed limit method overcomes these disabilities on processes having very high performance requirements. Decisions of the binary or yes/no type are made based on pseudo-limits more stringent than those specified. This will create pseudo-events that can be plotted to provide the desired commentary on the control chart. An instance is compressed limit gauging where gauges are set at inspection limits more stringent than specified limits. Compressed limits can be used, however, not only when examination is by gauging, but also whenever examination is by measurement, and a comparison of the measured results is made against a standard. Examples are check-weighing quantities or packages, or when checking such physical characteristics as flow properties, cross-breaking strength, viscosity, tensile strength and so on.

Case Study

An example of such a compressed limit application is in the control of automatic weighing and pressing of plastic moulding material into pellets.

In one cycle of this process, a granular plastics moulding material is fed, from an overhead hopper, through a reciprocating volumetric charge plate which releases the desired charge of material into a mould. In the next cycle, this charge of material is pressed into a pellet, for convenience of handling in the pre-heating and moulding process; this pellet is then ejected.

The weight of the pellet is to be held at 155 g ± 1.5 g and the output rate is approximately 6 000 pellets in eight hours.

Special sampling limits are set inside the specification limits of 153.5 g and 156.5 g, at 154.4 g and 155.6 g. These special sampling limits are based on knowledge of process setting and variability and are set to achieve, on average, a few "pseudo heavies" and "pseudo lights" in each sample. The weighing scale is clearly marked with the compressed limits. Since it is only necessary to scan the scale to see if a pellet weight falls outside the respective compressed limit weights, checking is done rapidly. A sample size of 20 is taken at appropriate intervals.

Such compressed limit attribute charts are operated in a similar manner as that described in **7.2.2**.

Annex A (normative)
Poisson and binomial tests

NOTE It is impracticable, within the scope of this standard, to provide methods of testing the Poisson or binomial distribution assumptions under all conditions. Some useful tests are described in this annex, but the reader is advised to refer to more comprehensive statistical texts for greater detail.

A.1 Test by comparison of observed and estimated theoretical variances

This is a simple test of dispersion. It compares the ratio of an observed variance with the theoretical variance to test whether it differs significantly from 1.

Step 1:

For g samples of binomial or Poisson observations, estimate:

$$\text{observed mean} = \overline{X} = \frac{1}{g}\sum X \tag{A.1}$$

$$\text{observed variance} = S^2 = \frac{1}{g-1}\left[\sum\left(X-\overline{X}\right)^2\right] \tag{A.2}$$

Step 2:

a) for Poisson distribution model:

theoretical variance [square of standard deviation, equation (4)] = m, the mean

where m is estimated from the data

Evaluate:

$$V = \frac{\text{observed variance}}{\text{estimated theoretical variance}} = \frac{S^2}{X} \tag{A.3}$$

b) for binomial distribution model:

theoretical variance = [square of standard deviation, equation (2)] = $np(1- p)$

where $np(1- p)$ is estimated from the data by substituting for the mean, np, thus:

$$\text{estimated theoretical variance} = np(1-p) = \frac{\overline{X}(n-\overline{X})}{n} \tag{A.4}$$

where n = number of items in each equal sized sample

Evaluate:

$$V = \frac{\text{observed variance}}{\text{estimated theoretical variance}} = \frac{nS^2}{\overline{X}(n-\overline{X})} \tag{A.5}$$

Step 3:

Assess the value of V in equation (A.3) or (A.5), as appropriate, against the entry for the number of samples, g, in Table A.1.

Step 4:

If V is between the critical values, there is no reason to suppose that the data does not reflect a Poisson or binomial distribution, as appropriate.

If V exceeds an upper critical value, there is evidence of variation in the average rate of occurrence of events, or in the proportion of attributes. This can arise, for example, from process fluctuations, from the occurrence of events in clusters rather than independently, or from variation in standards of assessment.

If V falls below a lower critical value, this can arise from some regular or systematic feature of the pattern of recurrence of events or attributes (rather than their random and independent occurrence).

Table A.1 — Critical values of variance ratio for testing binomial or Poisson distribution assumptions

Number of samples	Lower critical values		Upper critical values	
g	1 % significance	5 % significance	5 % significance	1 % significance
20	0.36	0.47	1.73	2.03
25	0.41	0.52	1.64	1.90
30	0.45	0.55	1.58	1.81
40	0.51	0.61	1.49	1.68
50	0.56	0.64	1.43	1.60
60	0.59	0.67	1.39	1.54
80	0.64	0.71	1.34	1.46
100	0.67	0.74	1.30	1.40
150	0.73	0.79	1.24	1.32
200	0.76	0.81	1.21	1.28

NOTE For other values of g, consult standard tables for the (chi-squared) χ^2 distribution. The values given in this table are the 0.5 %, 2.5 %, 97.5 % and 99.5 % points of $\frac{\chi^2}{g-1}$. Alternatively, use linear interpolation for intermediate values for g.

Step 5

If V is outside the critical values, further investigation is advisable to diagnose the reasons for this.

A.2 Successive differences test

A useful check on the independence of successive counts of attributes or events is provided by the successive differences test.

Step 1:

Estimate the standard deviation of the data by taking the square root of equation (A.2), thus:

$$S_1 = \sqrt{\frac{1}{g-1}\left[\sum\left(X-\overline{X}\right)^2\right]} \qquad \text{(A.6)}$$

Step 2:

Determine the standard deviation of the data from:

$$S_2 = \sqrt{\frac{1}{2(g-1)}\sum\delta^2} \qquad \text{(A.7)}$$

where δ^2 is the square of successive differences between counts of attributes or events

Step 3:

Divide (A.7) by the value obtained in (A.6), namely S_2/S_1, and evaluate this result against the limits:

$$1 \pm \frac{1}{\sqrt{(g+2)}} \qquad \text{(A.8)}$$

A value of S_2/S_1, $1-\frac{1}{\sqrt{(g+2)}}$, suggests a tendency for gradual shifts in the average rate, e.g. a trend or slow cycle.

A value of S_2/S_1 exceeding $1+\frac{1}{\sqrt{(g+2)}}$ suggests a tendency to alternate.

Annex B (normative) Poisson-based control chart limits

NOTE These are calculated on the same probability basis as standard measured data control charts.

Constancy of risk, at the level obtained with standard Shewhart-type measured data control limits, can usually readily be achieved using Poisson based control limits rather than the more generally used normal approximation to the Poisson. These Poisson-based control limits are set out for various means in this annex.

The tables in this annex are based on tail probabilities of 0.001 35 and 0.022 8 for the action and warning limits respectively. This corresponds with the same probabilities as ±3 standard deviations and ±2 standard deviations of the normal distribution used in measured data control charting.

It is not possible to have other than whole numbers in count data. For ease of visual clarity, limits are set a touch in from their threshold values. The value of 0.3 is used throughout.

Thus if an upper limit of 6 represents in-control but 7 represents out-of-control the upper limit is set just in from 7, namely, at 6.7. Similarly if 3, say, represents an out-of-control situation the lower control limit is set 0.3 in at 3.3.

Table B.1 — Poisson-based upper action control limits (set at 0.001 35 probability)

Upper action control limits													
mean	0.05	0.21	0.47	0.79	1.17	1.60	2.07	2.56	3.08	3.63	4.19	4.77	5.37
control	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙
limits	1.7	2.7	3.7	4.7	5.7	6.7	7.7	8.7	9.7	10.7	11.7	12.7	13.7
mean	5.38	5.98	6.60	7.23	7.88	8.53	9.19	9.86	10.54	11.23	11.92	12.62	13.32
control		↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙
limits		14.7	15.7	16.7	17.7	18.7	19.7	20.7	21.7	22.7	23.7	24.7	25.7
mean	13.33	14.03	14.75	15.47	16.19	16.92	17.70	18.43	19.18	19.92	20.67	21.43	22.18
control		↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙
limits		26.7	27.7	28.7	29.7	30.7	31.7	32.7	33.7	34.7	35.7	36.7	37.7
mean	22.19	22.94	23.71	24.47	25.24								
control		↙	↙	↙	↙								
limits		38.7	39.7	40.7	41.7								

EXAMPLE 1 Use of Table B.1. For a mean of 0.47, the upper action control limit is 3.7. For a mean between 0.48 and 0.79, the upper action control limit is 4.7.

Table B.2 — Poisson-based lower action control limits (set at 0.001 35 probability)

Lower action control limits												
mean	6.61	8.90	10.87	12.68	14.39	16.03	17.62	19.17	20.69	22.18	23.64	25.08
control	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘
limits	0.3	1.3	2.3	3.3	4.3	5.3	6.3	7.3	8.3	9.3	10.3	11.3

EXAMPLE 2 Use of Table B.2. For a mean of 10.87, the lower action control limit is 2.3. For a mean between 8.90 and 10.86, the lower action control limit is 1.3.

Table B.3 — Poisson-based upper warning control limits (set at 0.022 8 probability)

Upper warning control limits													
mean	0.02	0.23	0.60	1.06	1.58	2.15	2.76	3.39	4.05	4.72	5.41	6.11	6.83
control	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙
limits	0.7	1.7	2.7	3.7	4.7	5.7	6.7	7.7	8.7	9.7	10.7	11.7	12.7
mean	6.84	7.56	8.29	9.04	9.79	10.55	11.32	12.09	12.87	13.65	14.44	15.24	16.03
control		↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙
limits		13.7	14.7	15.7	16.7	17.7	18.7	19.7	20.7	21.7	22.7	23.7	24.7
mean	16.04	16.84	17.64	18.45	19.26	20.08	20.90	21.72	22.54	23.37	24.20	25.03	
control		↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	
limits		25.7	26.7	27.7	28.7	29.7	30.7	31.7	32.7	33.7	34.7	35.7	

EXAMPLE 3 Use of Table B.3. For a mean of 0.60, the upper warning control limit is 2.7. For a mean between 0.61 and 1.06, the upper warning control limit is 3.7.

Table B.4 — Poisson-based lower warning limits (set at 0.022 8 probability)

Lower warning control limits												
mean	3.78	5.68	7.34	8.90	10.38	11.82	13.22	14.59	15.93	17.26	18.57	19.86
control	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	
limits	0.3	1.3	2.3	3.3	4.3	5.3	6.3	7.3	8.3	9.3	10.3	
mean	19.87	21.16	22.43	23.69	24.95	26.20						
control	↘	↘	↘	↘	↘	↘						
limits	11.3	12.3	13.3	14.3	15.3	16.3						

EXAMPLE 4 Use of Table B.4. For a mean of 5.68, the lower warning control limit is 1.3. For a mean between 3.78 and 5.67, the lower warning control limit is 0.3.

blank

Bibliography

Standards publications

BS 5701-1, *Guide to quality control and performance improvement using qualitative (attribute) data — Part 1: Uses and value of attribute charts in business, industry, commerce and public service.*

BS 5701-2, *Guide to quality control and performance improvement using qualitative (attribute) data — Part 2: Fundamentals of standard attribute charting for monitoring, control and improvement.*

BS 5701-4, *Guide to quality control and performance improvement using qualitative (attribute) data — Part 4: Attribute inspection performance control and improvement.*

BS 5703 (all parts), *Guide to data analysis and quality control using cusum techniques.*

BSI — British Standards Institution

BSI is the independent national body responsible for preparing British Standards. It presents the UK view on standards in Europe and at the international level. It is incorporated by Royal Charter.

Revisions

British Standards are updated by amendment or revision. Users of British Standards should make sure that they possess the latest amendments or editions.

It is the constant aim of BSI to improve the quality of our products and services. We would be grateful if anyone finding an inaccuracy or ambiguity while using this British Standard would inform the Secretary of the technical committee responsible, the identity of which can be found on the inside front cover. Tel: +44 (0)20 8996 9000. Fax: +44 (0)20 8996 7400.

BSI offers members an individual updating service called PLUS which ensures that subscribers automatically receive the latest editions of standards.

Buying standards

Orders for all BSI, international and foreign standards publications should be addressed to Customer Services. Tel: +44 (0)20 8996 9001. Fax: +44 (0)20 8996 7001. Email: orders@bsi-global.com. Standards are also available from the BSI website at http://www.bsi-global.com.

In response to orders for international standards, it is BSI policy to supply the BSI implementation of those that have been published as British Standards, unless otherwise requested.

Information on standards

BSI provides a wide range of information on national, European and international standards through its Library and its Technical Help to Exporters Service. Various BSI electronic information services are also available which give details on all its products and services. Contact the Information Centre. Tel: +44 (0)20 8996 7111. Fax: +44 (0)20 8996 7048. Email: info@bsi-global.com.

Subscribing members of BSI are kept up to date with standards developments and receive substantial discounts on the purchase price of standards. For details of these and other benefits contact Membership Administration. Tel: +44 (0)20 8996 7002. Fax: +44 (0)20 8996 7001. Email: membership@bsi-global.com.

Information regarding online access to British Standards via British Standards Online can be found at http://www.bsi-global.com/bsonline.

Further information about BSI is available on the BSI website at http://www.bsi-global.com.

Copyright

Details and advice can be obtained from the Copyright & Licensing Manager. Tel: +44 (0)20 8996 7070. Fax: +44 (0)20 8996 7553. Email: copyright@bsi-global.com.

BSI
389 Chiswick High Road
London
W4 4AL